

# Aftershock triggering is a multi-step process

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# 1 Abstract

In most models of aftershock triggering, including static stress change plus rate and state friction, static fatigue, and stress corrosion, the mainshock stress change clock advances surrounding nucleation sources along pre-existing failure curves in a stress dependent manner. Therefore since the stress applied by the mainshock decreases rapidly with distance, average aftershock time should increase with distance, or aftershocks should appear to “diffuse” with time (Dieterich, 1994). Whether such diffusion actually occurs has been controversial, with some studies finding aftershock diffusion (Tajima and Kanamori, 1985; Henry and Das, 2001) and others finding that aftershock times do not significantly increase with distance (Gasperini and Mulargia, 1989; Shaw, 1993; Jones and Hauksson, 1998; Helmstetter et al., 2003). A critical difference between the two sets of studies is that the former generally assumes and corrects for a constant background seismicity rate while the latter implicitly assumes that there is no background during aftershock sequences. Here I avoid the issue of background rate correction by looking at only short times after California M 5–7 mainshocks, when the aftershock to background ratio is still quite high. Our results corroborate the previous findings that aftershock times do not increase with distance from the mainshock. This suggests that aftershock triggering is a multi-step process whereby the final determination of aftershock timing is independent of the initial triggering. The initial triggering step could be catastrophic weakening of the aftershock nucleation patch, with a probability that is stress-amplitude dependent, followed by forcing from postseismic slip, which has the same temporal decay rate as aftershock activity.

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# 2 Introduction

In most published physical models of aftershock triggering aftershocks occur when the stress change applied by the mainshock pushes earthquakes along their failure curves, leading to rapid earthquake clustering (see discussion in Gombert (2001)). Popular models of this type include rate and state friction plus static stress change (Dieterich, 1994), subcritical crack growth (Das and Scholz, 1981; Yamashita and Knopoff, 1987;

Shaw, 1993) and static fatigue (Scholz, 1968). I will term these “one state” models because the aftershock failure is driven by a simple one step clock advance of the nucleation processes along a pre-determined failure curve. A universal feature of one state models is that the time to rupture of a stressed source depends on the amplitude of the stress applied by the mainshock. Although individual sources will be advanced by different amounts depending on local initial conditions, on average the collections of sources that receive more stress will rupture more quickly (Dieterich, 1994; Huc and Main, 2003; Helmstetter et al., 2003). Since stress amplitude decreases with distance from the mainshock fault plane, this also means that there should be a positive correlation between distance from the mainshock and aftershock time, or that there should be an aftershock “diffusion”, as given by the equation,

$$R(t) \sim t^H \quad (1)$$

(Huc and Main, 2003) where  $t$  is time,  $H$  is a constant, and  $R$  may be the median aftershock distance as a function of time. For the static stress change plus rate and state friction model of Dieterich (1994)  $H \sim 0.3$  according to analysis by Huc and Main (2003).

Observations suggest, however, that contrary to these predictions aftershock timing does not correlate with distance to the mainshock. Gasperini and Mulargia (1989), for example, found no evidence of migration within aftershock sequences in Italy. Shaw (1993) found in a study of the aftershocks of 228 M 3–6 mainshocks occurring throughout northern and central California that the times and locations of the aftershocks were independent of each other. Jones and Hauksson (1998) found that aftershock density decreased with distance from the 1992  $M_W 7.3$  Landers rupture but that the rate of aftershock decay did not change. That is, considering the empirical relationship the modified Omori law (Utsu, 1961) that describes the decay of aftershock rate,  $R$ , with time ( $t$ ),

$$R(t) = \frac{K}{(t + c)^p} \quad (2)$$

the results of Jones and Hauksson (1998) gives that the parameters  $c$  and  $p$  remained constant while only  $K$  decreased with distance from the Landers rupture. In a survey of global earthquakes Huc and Main (2003) found aftershock diffusion to be very small, with  $H \sim 0.06$ . This agrees with the result of Helmstetter et al. (2003) who looked at 24 California aftershock sequences and found an average of  $H = 0.08$  (when aftershock distance was measured from the long elliptical axis of the aftershock sequence). Helmstetter

et al. (2003) concluded that aftershock diffusion is either very small or non-existent. Davidsen and Paczuski (2005) found no correlation between jumps in space and time between sequential catalog earthquakes in Southern California, which they concluded indicated that aftershocks decay at the same rate at all distances.

These papers are balanced by an extensive collection of work that claims that significant aftershock diffusion does occur (Tajima and Kanamori, 1985; Henry and Das, 2001; Marsan and Bean, 2003). In some cases aftershock diffusion may be seen because an earthquake occurs in special circumstances; Miller et al. (2004), for example, found very rapid diffusion in the 1997 Umbria-Marche sequence in Italy, which they postulated was driven by a known deep source of highly pressurized  $CO_2$ . This sequence was really a swarm, however, which is generally characterized by a steep uptick in the “background” seismicity rate driven by a local stressor (Llenos et al., 2009). In other cases aftershock diffusion may only appear to occur because of failure to correct adequately for the fact that the magnitude of completeness of aftershock catalogs increases with time as activity slows. When catalog completeness improves the aftershock zone appears to grow because so many more aftershocks are detected and mapped. For example, Felzer and Kilb (2009) demonstrated that when smaller aftershocks can be routinely detected aftershock zones appear to be much larger. This effect in particular may make aftershock zones appear to spread with time to the casual observer. Whether or not a study finds aftershock diffusion is also dependent on whether or not the assumption is made that all earthquake sources in the vicinity are affected by the mainshock, and thus become part of the aftershock sequence, or whether it is assumed that uncorrelated background earthquakes continue amongst the triggered aftershocks. As demonstrated by Helmstetter et al. (2003), if uncorrelated background earthquakes do continue to occur the mixing of these earthquakes with physically triggered aftershocks will create apparent aftershock diffusion whether physical diffusion occurs or not, because the average background event occurs both at later times and further distances than the average aftershock. When authors explicitly correct for both catalog completeness and background earthquakes, aftershock diffusion is found to be minimal or absent in all of the publications of which I am aware.

Here I seek to determine whether or not aftershock times vary with applied stress change without the requirement of making an assumption about background seismicity. This is done by inspecting only short times after the mainshock, since at short enough times aftershocks far exceed background seismicity. How short the time needs to be can be estimated from modeling, as described below. Obviously this method cannot elucidate what occurs at later times, but a finding of no correlation at short times tends to support

the same finding at later times and vice versa. I note that aftershock diffusion at short times has previously been described by Peng and Zhao (2009) for the 2004 Parkfield earthquake, however the diffusion found was small and episodic. The diffusion was also limited to the creeping section, and, most importantly, may at least in part be explained by an artifact of the detection method, as detailed in the supplementary material (Peng and Zhao (2009) Supplementary Figure S15 (a) and (b)). The statistical significance of the Parkfield diffusion was not determined.

In this paper I inspect for correlation between aftershock times and distances at short times following M 5–7 mainshocks in Southern California. I find that aftershock times do not show statistically significant variance with the stress applied by the mainshock. This result implies that the one state models do not explain aftershock triggering. Since the mainshock nonetheless clearly time advances the aftershocks, and since aftershock density clearly depends on distance from the mainshock fault, but the actual time of each aftershock is independent of the amount of stress applied, I infer that aftershock triggering must be a multi-step process which allows for decoupling between the initial triggering, which occurs with a stress amplitude-dependent probability, and the determination of aftershock rupture time. In particular, I hypothesize that the mainshock initially only changes the frictional parameters or fault properties at the future aftershock hypocenter, paving the way for other processes to occur. I refer to this as “mutli-step” aftershock triggering. What such a model might look like is discussed further below and in a companion publication.

### 3 Data Analysis

#### 3.1 Data selection and processing

I use M 5–7 mainshocks from the relocated Southern California 1984–2002 earthquake catalog of Shearer et al. (2003) to verify whether or not aftershock timing varies with distance from the mainshock. Analysis of larger earthquakes becomes more complicated as the location of the extended fault plane becomes more important; however, as noted above, an individual assessment of the 1992 M 7.3 Landers earthquake sequence was performed by Jones and Hauksson (1998). Earthquakes  $M < 5$  are not used because distances within one fault length are within the range of aftershock location error. I also only use aftershocks that are 0.1 to 2 mainshock fault lengths away from the mainshock fault. The outer distance limit is chosen to correspond with the furthest distance for which static stress change may trigger aftershocks with statistical significance

(Hardebeck et al., 1998), while the inner limit is chosen because of limited stress change information and aftershock location error at closer distances. King et al. (1994), for example, excluded seismicity within 5 km of the Landers fault plane from static stress change analysis, which is  $\sim 0.1$  fault lengths of that earthquake.

To ensure complete aftershock sequences I only use aftershocks that have magnitudes greater than 2 or  $M_{main} - 2.5$ , whichever is larger. These cutoffs are chosen following the guidelines of Helmstetter et al. (2005) for aftershock catalog completeness in California after the first 60 seconds, during which time I expect many aftershocks to be lost in the mainshock coda (Kilb et al., 2007). Thus all data that I use, and all simulations that I run, as described below, exclude the first 60 seconds after the mainshock. To verify the completeness of the aftershocks that I use I verify that there is no significant correlation between aftershock magnitude and time.

I do not use all M 5–7 earthquakes in the catalog as mainshocks because I wish to avoid using mainshocks that occur so close in time and space to larger earthquakes that aftershocks of the latter may be erroneously assigned as aftershocks of the former. As described above I also need to limit the observation time of aftershocks to minimize interference from background seismicity, if it exists. With these goals in mind I only use earthquakes as mainshocks if they are separated from larger earthquakes by at least  $L$  fault lengths or  $t_1$  days if the larger earthquake comes first and  $t_2$  days if the larger earthquake comes second, where aftershocks are gathered for  $t_2$  days. To find the optimal values of these parameters I use a stochastic ETAS (Epidemic Type Aftershock Sequence) algorithm to simulate several southern California earthquake catalogs made of aftershock sequences and background seismicity. Details of the ETAS simulation, including the parameters used, are given in Hardebeck et al. (2008). The implementation used here differs from Hardebeck et al. (2008) in only that a more precise background seismicity grid is used; see the discussion and description in the Appendix. The ETAS simulations contain no built-in coupling between aftershock time and location. Thus if significant correlation is seen in aftershocks gathered from the simulation it means that  $L$  and/or  $t_1$  are too small or  $t_2$  is too long, such that too many background earthquakes or aftershocks of other events are erroneously being assigned to target mainshocks. Using a grid search, I find that one combination of parameters that both keeps the correlation coefficient low enough and preserves enough data for one step models to be rejected at 95% confidence are  $L = 10$  fault lengths,  $t_1 = 8$  days, and  $t_2 = 0.5$  days. Details on how I test whether a correlation coefficient is low enough to reject the one step model are given below.

The final step before I can compare aftershock times and distances to the fault plane is estimating the location of the mainshock fault plane. I estimate the length and width of the fault planes using the empirical magnitude scaling relationships of Wells and Coppersmith (1995). I then take all of the earthquakes occurring within 0.25 days and one fault length of each mainshock, which should primarily be aftershocks in accordance with our previous analysis (e.g.  $t_2 = 0.25$  days). The center of the mainshock fault plane is set to the median location of these aftershocks. Fault strike is found from the best fitting (linear least squares) line to these aftershocks in the x-y plane; the fault is then rotated so that its strike aligns with the y axis, and dip is resolved by fitting the locations of the rotated aftershocks in the vertical plane. When less than five aftershocks are present a fault plane cannot be fit, and the mainshock is approximated by a point source at the median aftershock location. This method assumes that the mainshock occurs on a single rectangular plane with a uniform strike and dip, which is not always the case. A number of earthquakes may be approximated as being planar, however, at least on the kilometer scale, and this approximation is common in the literature. Most importantly, measuring aftershock distances to the mainshock faults thus defined produces aftershock densities that clearly decay with distance (the Spearman’s correlation coefficient between distance and density is  $r = -0.8$ ) (Figure 1). The goal with measuring the distances is not that the distances be precise but that the distance can be used as an effective proxy for the stress amplitude induced by the mainshock. The sharp drop of earthquake density with the measured distance indicates that the measurement is indeed an effective proxy.

## 4 Results

The rules for mainshock and aftershock selection given in the previous section allow for the use of 45 M 5–7 mainshocks and 301 aftershocks the latter of which, as noted above, all occur within 0.1-2 fault lengths and 0.5 days of their mainshocks. The Spearman’s correlation coefficient for time vs. distance from the mainshock fault for these aftershocks is  $r = 0.088$  ( $r^2 = 0.008$ ) which is not statistically significant at 95% confidence, although it’s very close ( $p = 0.06$ ). This by itself does not say much, however, as it is always possible that with the addition of more data statistical significance would be obtained. We also know that both background seismicity and secondary aftershock clusters will always create some correlation, as described previously (Helmstetter et al., 2003), independent of the triggering mechanism. As also described

in the previous section, however, the mainshock and aftershock parameters have been set such that I should be able to test if the correlation coefficient in the data is statistically lower than what would be expected in the case of one-step triggering. I describe next how I estimate what correlation coefficients one-step triggering models should produce.

Any accurate aftershock triggering mechanism must reproduce the robustly observed Omori's law or the modified Omori law (Equation 2). This law has been found to hold out to distances of at least two fault lengths from the mainshock fault. Therefore changes in aftershock behavior with distance must manifest as changes in either the  $K$ ,  $c$ , or  $p$  parameters of the law with distance, not wholesale violation of the relationship. Dieterich (1994) specifically states that static stress change plus rate and state friction triggering results in an increase of the  $c$  parameter with distance while the other modified Omori law parameters remain constant. Should all one step models show the same behavior, or are changes in the other parameters possible?

Changing the  $K$  parameter by itself affects the total number of aftershocks but does not affect aftershock time, therefore changing  $K$  only with stress amplitude would not result in a change of temporal distribution with distance, contrary to the one step models. In fact any change in  $K$  with stress, even if  $c$  and/or  $p$  are changing at the same time, leads to some degree of change in earthquake rate with stress that is decoupled from changes in the earthquake temporal distribution. In one step models differences in stress change are always coupled with changes in failure time, and it is this dependency that is completely responsible for changes in total earthquake rate with applied stress. Therefore changes in  $K$  as a function of stress amplitude are not supported by these models.

Change in the  $p$  parameter produces change in aftershock times, however it produces contrary results at different times; a higher  $p$  value produces more earthquakes at short times and less at longer times, and vice versa. Thus a decrease of  $p$  value with applied stress would produce faster aftershocks with higher stress loads in the short term, but slower aftershocks at higher stress loads at later times, which is inconsistent with the one step models in which larger stresses always result in more rapid rupture. This effect might be countered if near and far  $p$  values underwent changes of opposite sign with time. This would require a very complex physical model, however, and I am not aware of any of this type that has been published to date. In the leading one step models time to failure varies exponentially with the stress; applying a uniform stress step to earthquakes on this failure curve results in failure times that follow the modified Omori law with

$p = 1$  regardless of stress change amplitude. It appears reasonable, therefore, that for all one step models only the  $c$  parameter should change with stress amplitude.

I use my aftershock data set described above to calculate empirically how  $c$  should change with distance. I first combine the aftershocks into a single effective aftershock sequence, as described in Felzer et al. (2002). I then set the  $p$  value to 1.08 (Reasenberg and Jones, 1989; Felzer et al., 2003) and solve for values of  $K$  and  $c$ , balancing the tradeoff between these parameters with the constraint that the average  $c$  over all distances should come close as possible to the average  $c$  of 0.05 days found by Reasenberg and Jones (1989) for California aftershock sequences. I find that a solution in which the mean  $c$  value is exactly  $= 0.05$  days cannot fit the aftershock rates in time and space; the lowest mean value of  $c$  that does work is  $c = 0.21$  days. With this solution, however, the values of  $c$  at most distances are still quite small and the median value of  $c$  is only 0.02 days. The variation of  $c$  with distance is given in Figure 1. The corresponding value of  $K$ , averaged over the different mainshock magnitudes and minimum aftershock magnitudes, is 32.9.

With the values of  $c$ ,  $K$ , and  $p$  determined, I next produce 5000 Monte Carlo simulated data sets of aftershocks that are consistent with one step model triggering. For the modeling for each aftershock I first assign a distance that falls randomly between the distance of two aftershocks from the real data set, and then randomly assign a time from the modified Omori law, using the empirically found values of  $K$ ,  $p$ , and distance-dependent  $c$ . The decay in time and space of each set of simulated aftershocks matches closely with the overall temporal and spatial decay seen in the real data; an example of a random set of simulated data plotted against real data in the temporal domain is given in Figure 2.

The Spearman's correlation coefficients for aftershock distance vs. time measured from each simulation of one step triggering model aftershocks is given in Figure 3, along with the correlation coefficient measured from the real data. The mean correlation coefficient from the simulated data is 0.44, with  $\sigma = 0.05$ . Therefore, if I approximate the simulation results with a normal distribution, the correlation coefficient measured from the real data ( $r = 0.088$ ) allows us to reject the one step triggering models at  $> 99\%$  confidence ( $p = 9 \times 10^{-13}$ ). Thus not only is the temporal/spatial correlation of real aftershock data low, but it is far below what would be expected in any model in which aftershock triggering is accomplished by the simple clock advance of earthquakes along a pre-determined failure curve. Huc and Main (2003) claimed that the low value of aftershock diffusion that they found could be fit with a subcritical crack growth triggering model, but they obtained their model parameters from laboratory values rather than by

fitting observed aftershock data.

## 5 Discussion

The results above indicate that aftershocks are not triggered by simple permanent clock advances along a pre-existing failure curve. The failure of the one step models triggering is perplexing because these models are laboratory-supported, simple, and physically compelling. I postulate that the one step models may not work in the real earth because the stress on faults is usually so far below their static strength that mainshock-applied stress is not sufficient to push a large number of sources far enough along their failure curves. I note that stresses are much higher and strength much lower during dynamic rupture than during static conditions. Thus if static stress was normally close to static strength along vast areas of active faults it would be difficult to understand why most majority of earthquakes stop when they are still quite small. This problem is often solved by assuming that the points where earthquakes nucleate have much higher initial stress than the rest of the fault. But it is possible that stress contrasts this large on the fault may either not exist or simply not be sufficiently stable to last over time. Instead, the nucleation site of most earthquakes may be where the fault has been made much weaker than the surrounding area by the actions of a previous earthquake.

I thus postulate that when seismic waves interact with the fault they may with some probability lead to a catastrophic process in which a small part of the fault is severely weakened. After this weakening the point could be pushed to nucleation by a separate local process such as post-seismic afterslip, which has been shown to decay in the same manner as aftershocks (Perfettini and Avouac, 2004; Hsu et al., 2006). Because this triggering model requires that the fault be transformed from a strong to a severely weakened step as part of the triggering process, and because two different steps (weakening and then aseismic forcing) are involved, I term this model two-step or multi-step aftershock triggering. Further details on the potential physics of this mechanism and additional observational support will be given in a companion paper.

It is true that our data here covers only the first 12 hours of the aftershock sequence; I kept our observation time short so that I would not need to make assumptions about background seismicity. As a result our results do not contradict that one-step triggering occurs at longer times. There are no observations, however, in the form of discontinuities in decay curves or other aspects of aftershock behavior that suggests that there is a widespread change in the aftershock triggering mechanism with time. Therefore our observation here

that mainshock stress change amplitudes do not significantly affect aftershock times in the first 12 hours support the findings of the previous authors who found that the correlation does not exist at later times.

## 6 Conclusions

In the vast majority of physical aftershock triggering models, including static stress change plus rate and step friction, subcritical crack growth, and static fatigue, triggering occurs when the mainshock-applied stress moves earthquake sources forwards along a pre-existing failure curve. This process requires that earthquakes that receive more stress fail, on average, more quickly. In this study I inspect the first twelve hours of the aftershock sequences of 45 M 5–7 mainshocks occurring in Southern California which were relatively isolated in space and time from larger earthquakes. The short time allows us to avoid most background seismicity, thus avoiding the need to make assumptions about it, one of the key issues in previous studies on this topic. I find the correlation between distance from the mainshock fault and aftershock time to be very small, and to be significantly below what I would expect in the case of one step triggering ( $p = 9 \times 10^{-13}$ ). This result does not mean that mainshock stress amplitude has no importance in aftershock triggering; aftershock density clearly decreases rapidly with distance from the mainshock fault. Instead I suggest that the results indicate that aftershock triggering must be a multi-step process in which the initial triggering, which occurs with a probability that is stress amplitude dependent, and determination of final rupture time are independent of each other. Specifically I suggest that dynamic stress might initiate a process that leads to catastrophic weakening on small patches of the fault, followed by forcing by postseismic slip that determines the actual failure time.

## 7 Data and Resources

The primary earthquake data used for this study came from the relocated southern California catalog of Shearer et al. (2003). Additional catalog data used for the background matrix of the ETAS earthquake and aftershock simulations was from the Southern California Seismic Network (SCSN) and was obtained from the web page [http://www.data.scec.org/catalog\\_search/date\\_mag\\_loc.php](http://www.data.scec.org/catalog_search/date_mag_loc.php), last accessed on September 5,

2008.

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## Figures

**Figure 1** Panel (a) gives aftershock density as a function of mainshock fault lengths away from the mainshock fault plane. Aftershock density is linear density per km measured between every two earthquakes via the nearest neighbor method (e.g. see (Felzer and Brodsky, 2006)). Panel (b) gives the  $c$  values required to fit the observed aftershock density decay with distance if  $c$  is the only parameter in Omori's law that varies with distance.

**Figure 2** This figure gives aftershock rate, in units of number of aftershocks per 30 minutes, for the real data (solid black line) and data simulated from the one state triggering model in which the modified Omori law  $c$  value increases with distance (dashed black line).

**Figure 3** The non-parametric (Spearman's) correlation coefficient between aftershock times and distance. The correlation coefficient for the data is given by the dashed black line and is calculated from 45 M 5–7 mainshocks and 301 aftershocks occurring within 12 hours and 0.1 to 2 fault lengths. The gray histogram gives correlation coefficients calculated from 5000 Monte Carlo simulations of one state type aftershock triggering models (see text).

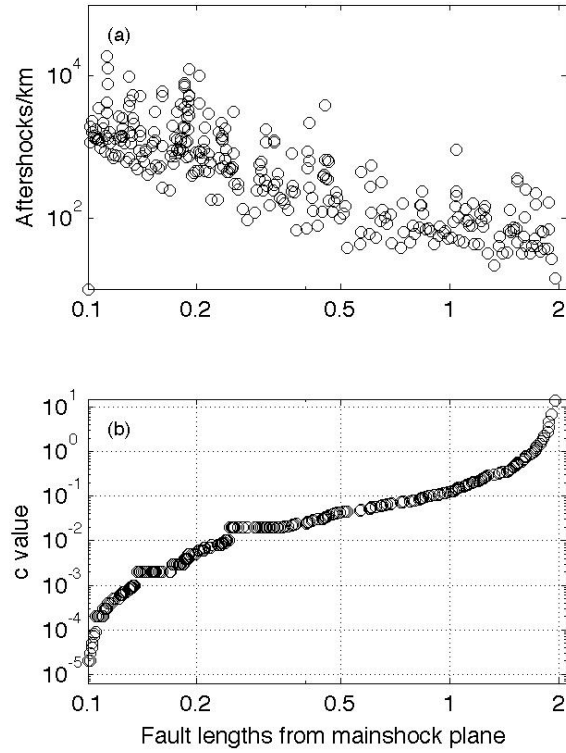


Figure 1: Panel (a) gives aftershock density as a function of mainshock fault lengths away from the mainshock fault plane. Aftershock density is linear density per km measured between every two earthquakes via the nearest neighbor method (e.g. see (Felzer and Brodsky, 2006)). Panel (b) gives the  $c$  values required to fit the observed aftershock density decay with distance if  $c$  is the only parameter in Omori's law that varies with distance.

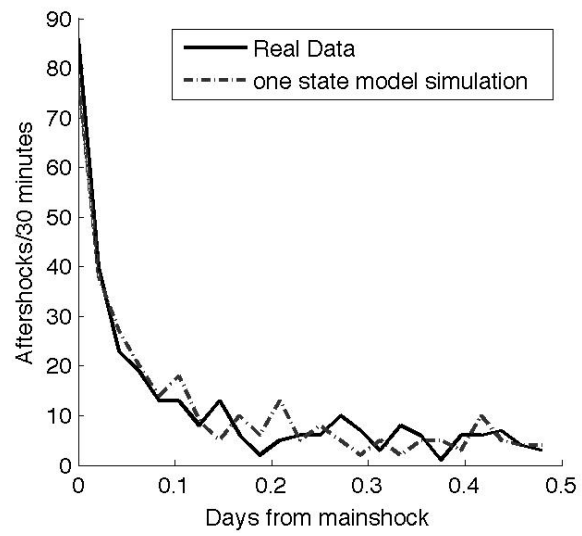


Figure 2: This figure gives aftershock rate, in units of number of aftershocks per 30 minutes, for the real data (solid black line) and data simulated from the one state triggering model in which the modified Omori law  $c$  value increases with distance (dashed black line).

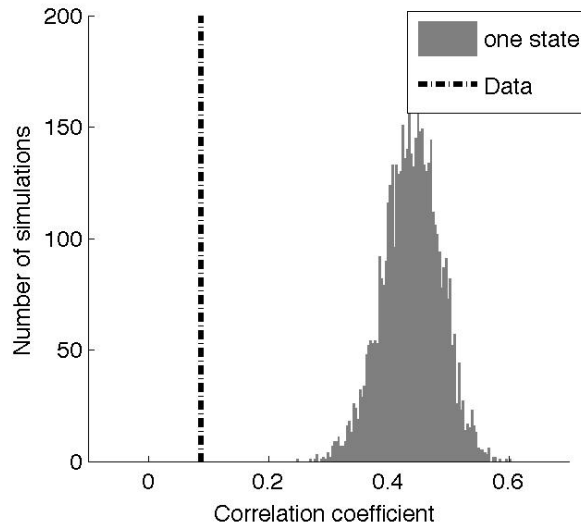


Figure 3: The non-parametric (Spearman's) correlation coefficient between aftershock times and distance. The correlation coefficient for the data is given by the dashed black line and is calculated from 45 M 5–7 mainshocks and 301 aftershocks occurring within 12 hours and 0.1 to 2 fault lengths. The gray histogram gives correlation coefficients calculated from 5000 Monte Carlo simulations of one state type aftershock triggering models (see text).

## Appendix A: The background seismicity matrix for the ETAS aftershock simulation

The background seismicity portion of an ETAS simulation consists of a steady, Poissonian seismicity rate with spatially varying density. Ideally the background seismic density reflects the rate of truly independent, steady state Poissonian earthquakes (if such events actually exist!).

The difficulty is that it is impossible to delineate which earthquakes in the real catalog are truly independent, especially when many events may be aftershocks of mainshocks too old or too small to appear in the catalog. Furthermore, if I cannot directly model the aftershocks of old and small mainshocks then I want to include such aftershocks in the modeled background, even if they are not technically “background”, so that they are not absent from the simulation.

The degree of spatial clustering in the background seismicity model influences the correlation of aftershock times and distances within ETAS simulated aftershock sequences. The background seismicity kernel of Hardebeck et al. (1998) was based on a sophisticated declustering method, but the catalog was not long enough to assign accurate background rates to small areas. One quick and dirty solution for making an adequately spatially clustered background seismicity kernel is to simply divide the region into small grid cells and take the average seismicity rate in each cell, after excluding time periods of high aftershock activity, which I define as seismicity in an  $N$  by  $N$  box occurring at a rate of  $> M$  earthquakes/day, where  $M$  is set such that the summed background will be 40% of the total Southern California seismicity rate, as found by Gardner and Knopoff (1974). I want to make the boxes as small as possible without hitting the scale of earthquake location error, but when  $N$  is made too small so many boxes contain only 0 or 1 earthquake that it is no longer possible to find a working value of  $M$ . The smallest  $N$  that I found would work was 0.9 degrees, which corresponds to  $0.5 M \geq 2$  earthquakes/day to produce a 40% background rate.

Note that since aftershock sequences decay with time  $t$  as  $1/t$  a few aftershocks will continue at a low nearly constant rate for a very long period of time and thus for the purposes of this paper become part of the background seismicity – that is, at long times the aftershocks constitute a distribution of earthquakes, occurring at a relatively constant rate, that are independent of the aftershock sequence of a recent mainshock of interest and may interfere with efforts to make statistical measurements of those aftershocks. With our method of forming the background grid, however, the aftershocks of large events that occur during the time

of interest – for example the aftershocks of the 1992  $M7.3$  mainshocks that occurred in the middle of our catalog – will be modeled as background at all times, not just after 1992. This is expected to produce a background grid that is more dispersive than it really was prior to 1992. This effect also operates on a smaller scale for many other mainshocks. This is the probable cause of our observation that aftershock time-space correlation seen in the ETAS simulations tends to be somewhat higher than the correlation seen in real data. This higher correlation in simulated seismicity was also found by Helmstetter et al. (2003).